SOLUTION OF INVERSE PROBLEMS IN HEAT TRANSFER BY THE METHOD OF TRANSITION FUNCTIONS

B. I. Strikitsa

Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 3, pp. 330–335, 1967

UDC 536.2.01

A method has been developed for determining the transition function Φ for transverse flow around a cylinder.

Experimental data [1] on the local heat-transfer coefficients in convective heat transfer with Re = $= 21 \cdot 10^3$ have been used to develop the method using the functions Φ. The solution obtained in this way is compared with the analytic solution of [1]. The problem is handled as follows.

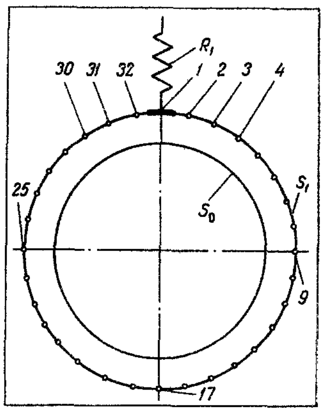

Fig. 1. Electrical model of a cylinder used to find Φ.

1. The cylindrical region G is specified.
2. The temperature t(x, y) within the region satisfies

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0. \qquad (1)$$

3. A constant temperature t_0 is maintained on the internal surface S_0.
4. The experimentally determined temperature distribution is used for the external surface S_1.
5. The temperature of the incident air flow is given.

The problem is to determine the local heat-transfer coefficients α_{loc}.

The problem is solved in three stages. First, an electrical analog is used to determine the transition functions Φ by a method analogous to that for determining the Z functions considered in [2, 3].

A two-layer model on the scale m* = 8 (Fig. 1) is constructed from conducting paper; the error in the solutions given by such models does not exceed 1.5% [4]. The potential equal to 0% is applied to the internal surface, while the external surface is divided into 32 equal parts; the numbers 1, 2, ..., 32 in Fig. 1 denote these parts. A current-carrying lead is applied to one of these parts; the current I_1 is determined from the potential difference across a resistor R_1 connected to the lead. The potential difference on R_1 is adjusted to 100%. The potentials U_i at the points 1, 2, ..., 32 are measured, the transition functions being defined as

$$\Phi_{i1} = U_i/I_1. \qquad (2)$$

The Φ_{i1} are sufficient to solve the inverse heat-transfer problem for an axially symmetric body, but all the Φ_{ij} must be known for a body of arbitrary shape, in which i are the points at which the U_i are measured and j are the points at which the currents I_j are supplied in turn.

In the second stage, a system of linear algebraic equations is compiled, whose solution gives the local currents at the surface of the model.

If currents are supplied simultaneously to the 32 points on the outer surface, the potentials at those points are found from the principle of superposition, which applies to linear systems:

$$\sum_{j=1}^{n} \Phi_{ij} \cdot I_j = U_i, \qquad (3)$$

in which i = 1, 2, ..., m and j = 1, 2, ..., n.

We must have m = n if the solution to this system is to be unique (m = n = 32 in our case); here the I_j act as unknowns, while the U_i are readily determined from the temperature distribution via a scale coefficient C_t, which is taken as 1. The choice of scale coefficients is described below.

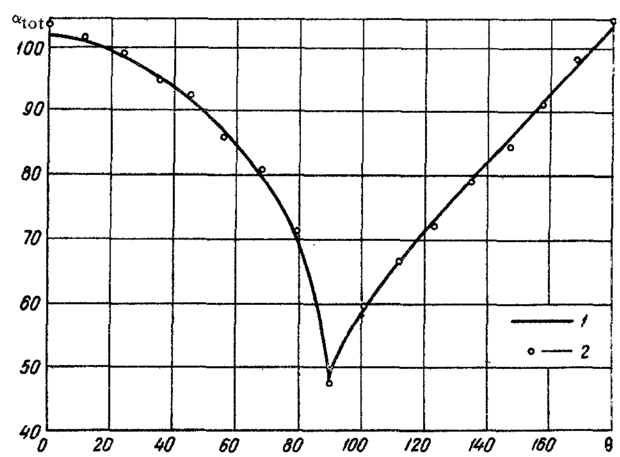

Fig. 2. Distribution of the heat-transfer coefficient α_{tot} (W/m^2 · deg) as a function of θ (deg) for the outer surface of a sphere: 1) data of [1], 2) results from use of Φ.

The result is a system of 32 linear algebraic equations in 32 unknowns, which is represented symbolically [5] in Table 1. Computer solution gives I_{loc}, the currents at the surface of the model (Table 2), which

Table 1

System of 32 Linear Algebraic Equations in 32 Unknowns

№ \ I_j	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}	I_{12}	I_{13}	I_{14}	I_{15}	I_{16}	I_{17}	U_i
1	11954	3914	1146	347	99	33	11	0	0	0	0	0	0	0	0	0	0	27.4
2	3914	11954	3914	1146	347	99	33	11	0									27.1
3	1146	3914	11954	3914	1146	347	99	33	11	0								26.6
4	347	1146	3914	11954	3914	1146	347	99	33	11	0							26.0
5	99	347	1146	3914	11954	3914	1146	347	99	33	11	0						25.4
6	33	99	347	1146	3914	11954	3914	1146	347	99	33	11	0					24.3
7	11	33	99	347	1146	3914	11954	3914	1146	347	99	33	11	0				23.0
8	0	11	33	99	347	1146	3914	11954	3914	1146	347	99	33	11	0			20.8
9	0	0	11	33	99	347	1146	3914	11954	3914	1146	347	99	33	11	0		17.9
10	0		0	11	33	99	347	1146	3914	11954	3914	1146	347	99	33	11	0	18.9
11	0			0	11	33	99	347	1146	3914	11954	3914	1146	347	99	33	11	20.2
12	0				0	11	33	99	347	1146	3914	11954	3914	1146	347	99	33	21.4
13	0					0	11	33	99	347	1146	3914	11954	3914	1146	347	99	22.8
14	0						0	11	33	99	347	1146	3914	11954	3914	1146	347	24.0
15	0							0	11	33	99	347	1146	3914	11954	3914	1146	25.3
16	0								0	11	33	99	347	1146	3914	11954	3914	26.5
17	0									0	11	33	99	347	1146	3914	11954	27.2
18	0										0	11	33	99	347	1146	3914	26.5
19	0											0	11	33	99	347	1146	25.3
20	0												0	11	33	99	347	24.0
21	0													0	11	33	99	22.8
22	0														0	11	33	21.4
23	0															0	11	20.2
24	0																0	18.9
25	0																	17.9
26	0																	20.8
27	11	0																23.0
28	33	11	0															24.3
29	99	33	11	0														25.4
30	347	99	33	11	0													26.0
31	1146	347	99	33	11	0												26.6
32	3914	1146	347	99	33	11	0	0	0	0	0	0	0	0	0	0	0	27.1

Table 1 (cont'd)

№ \ I_j	I_{18}	I_{19}	I_{20}	I_{21}	I_{22}	I_{23}	I_{24}	I_{25}	I_{26}	I_{27}	I_{28}	I_{29}	I_{30}	I_{31}	I_{32}	U_i
1	0	0	0	0	0	0	0	0	0	11	33	99	347	1146	3914	27.4
2										0	11	33	99	347	1146	27.1
3											0	11	33	99	347	26.6
4												0	11	33	99	26.0
5													0	11	33	25.4
6														0	11	24.3
7															0	23.0
8															0	20.8
9															0	17.9
10															0	18.9
11	0														0	20.2
12	11	0													0	21.4
13	33	11	0												0	22.8
14	99	33	11	0											0	24.0
15	347	99	33	11	0										0	25.3
16	1146	347	99	33	11	0									0	26.5
17	3914	1146	347	99	33	11	0								0	27.2
18	11954	3914	1146	347	99	33	11	0							0	26.5
19	3914	11954	3914	1146	347	99	33	11	0						0	25.3
20	1146	3914	11954	3914	1146	347	99	33	11	0					0	24.0
21	347	1146	3914	11954	3914	1146	347	99	33	11	0				0	22.8
22	99	347	1146	3914	11954	3914	1146	347	99	33	11	0			0	21.4
23	33	99	347	1146	3914	11954	3914	1146	347	99	33	11	0		0	20.2
24	11	33	99	347	1146	3914	11954	3914	1146	347	99	33	11	0	0	18.9
25	0	11	33	99	347	1146	3914	11954	3914	1146	347	99	33	11	0	17.9
26		0	11	33	99	347	1146	3914	11954	3914	1146	347	99	33	11	20.8
27			0	11	33	99	347	1146	3914	11954	3914	1146	347	99	33	23.0
28				0	11	33	99	347	1146	3914	11954	3914	1146	347	99	24.3
29					0	11	33	99	347	1146	3914	11954	3914	1146	347	25.4
30						0	11	33	99	347	1146	3914	11954	3914	1146	26.0
31							0	11	33	99	347	1146	3914	11954	3914	26.6
32	0	0	0	0	0	0	0	0	11	33	99	347	1146	3914	11954	27.1

Table 2
Distribution of the Principal Parameters on the External Surfaces of Model and Cylinder

Point no. in Fig. 1	θ(deg) in Fig. 1	Point no. in Fig. 1	θ(deg) in Fig. 1	$I_{loc} \cdot 10^6$, A	$k \cdot 10^6$, V/m²	$q_{loc} = kI$, W/m²	$\Delta t_{loc} = t_s - t_{int}$, °C	$\alpha_{loc.tot} = q/\Delta t$, W/m²·deg	$\alpha_{tot} = \alpha_c + \alpha_r$, W/m²·deg from [1]
1	0			1205	4.2	5061.0	48.4	104.6	102.0
2	11.25	32	348.75	1182	4.2	4964.4	48.7	101.9	100.2
3	22.5	31	337.5	1156	4.2	4855.2	49.2	98.6	97.9
4	33.75	30	326.25	1127	4.2	4733.4	49.8	95.1	95.6
5	45	29	315	1116	4.2	4687.2	50.4	93.0	92.2
6	56.25	28	303.75	1059	4.2	4447.8	51.5	86.4	87.0
7	67.5	27	292.5	1023	4.2	4296.6	52.8	81.4	79.7
8	78.75	26	281.25	926	4.2	3889.2	55.0	70.7	69.5
9	90	25	270	662	4.2	2780.4	57.9	48.0	49.1
10	101.25	24	258.75	815	4.2	3423.0	56.9	60.2	59.8
11	112.5	23	247.5	879	4.2	3691.8	55.6	66.4	66.0
12	123.75	22	236.25	922	4.2	3872.4	54.4	71.8	72.5
13	135	21	225	995	4.2	4179.0	53.0	78.9	79.1
14	146.25	20	213.75	1038	4.2	4359.6	51.8	84.2	85.6
15	157.5	19	202.5	1100	4.2	4620.0	50.5	91.5	91.6
16	168.75	18	191.25	1162	4.2	4880.4	49.3	99.0	97.6
17	180			1220	4.2	5124.0	48.6	105.4	104.1

correspond to the distribution of Q_{loc}, the heat flux at the surface of the cylinder. We employ scale coefficients to convert from the electrical quantities to the thermal ones, which coefficients must obey

$$C_R \frac{C_Q}{C_t} = 1, \qquad (4)$$

in which $C_R = R_e/R_t$ is the ratio of the electrical resistance of a sector cut from the model to the thermal resistance of the corresponding sector of the cylinder and $C_t = (U - U_0)/(t - t_0)$ is taken as 1, with the origins taken as the potential and temperature of the internal surface, respectively, while $C_Q = I/Q$ is determined from (4).

As $R_e = 23\,000$ om, $R_t = 1.27$ deg/W, and $C_R = 18\,110$ ohms · W/deg,

$$C_Q = C_t/C_R = 1/18110 \text{ V}^{-1}. \qquad (5)$$

Then

$$Q_{loc} = I_{loc}/C_Q = 18110\, I_{loc} \quad \text{W}. \qquad (6)$$

In the third stage we deduce the α_{loc} as

$$\alpha_{loc} = q_{loc}/\Delta t_{loc}. \qquad (7)$$

Then

$$q_{loc} = Q_{loc}/F_t, \qquad (8)$$

in which $F_t = B_t$ is the area of a part of the cylinder of arc length B_t and unit height. Here

$$B_t = B_e/m^*, \qquad (9)$$

in which B_e is the length of the outer circle of the model used in supplying current in the determination of Φ.

From (6)–(9) we have

$$q_{loc} = I_{loc}\, k = 4.2\, I_{loc} \quad \text{W/m}^2, \qquad (10)$$

in which

$$k = C_R m^*/C_t B_e \;\; \text{V/m}^2. \qquad (11)$$

Table 2 gives the numerical values of q_{loc}; Table 2 and Fig. 2 compare the total local transfer coefficients $\alpha_{loc.tot}$ as found analytically [1] and via the Φ. The discrepancies do not exceed 3%.

A major advantage of this method is that Φ once found on the model can subsequently be used to solve problems in the absence of the electrical model. This is of considerable importance, since any given object can give rise to many inverse problems, whose solution is much facilitated by the Φ.

A further advantage is that the Φ may be used to solve inverse problems in steady-state heat transfer for all similar bodies, the only change being recalculation of the scale coefficient k via (11).

An analogous method is readily developed for inverse problems in transient-state heat transfer, values of the Φ being obtained for each point in time. The method is also applicable to bodies of any shape.

NOTATION

I is current, U is voltage, R_e is electrical resistance, B_e is 1/32 of the length of the external circumference of the model, m* is the scale of model, k is the coefficient for converting from current to heat flux density, t is the temperature, t_0 is the temperature of internal surface of cylinder, x and y are coordinates, Q is heat flux, q is the density of heat flux, α is the heat-transfer coefficient, $\alpha_{tot} = \alpha_c + \alpha_r$, α_c is the convective heat-transfer coefficient, α_r is the radiative heat transfer coefficient, R_t is thermal resistance, Δt is temperature difference, B_t is 1/32 of length of external circumference of cylinder.

REFERENCES

1. G. N. Kruzhilin and V. A. Shvab, ZhTF, 5, no. 4, 1935.

2. L. I. Gutenmakher, Electrical Analogs [in Russian], Izd. AN SSSR, 1949.

3. L. I. Gutenmakher, Foreword to the Russian edition of Carpuls's Analog Devices for Solution of Problems in Field Theory, IL, 1962.

4. L. A. Kozdoba and L. V. Knyazev, collection: Theory and Applications of Mathematical Analogs [in Russian], Izd. Sov. radio, Moscow, 1965.

5. K. Lanczos, Practical Methods of Applied Analysis [Russian translation], GIFML, 1961.

18 July 1966

Naval Engineers Institute, Odessa